Supplemental Material for “Spatially shaping waves to penetrate deep inside a forbidden gap”

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(Dated: 21 July 2020)

In this supplementary material, we discuss how we determine the extinction length in the 2D photonic crystals. To this end, we first discuss the spatial resolution of the setup that is an important ingredient in the estimation of the extinction length.

SPATIAL RESOLUTION OF LATERAL SCATTERED LIGHT

Structural disorder is known to result in a new length scale for wave transport called the extinction length $\ell_{\text{ext}}$ that statistically quantifies the strength of the disorder [32, 33]. The extinction length of light in the two-dimensional silicon photonic crystal was characterized by imaging the $yz$–plane of the sample on an InGaAs camera with an effective optical magnification of 125×. A constant phase of 0 rad was displayed on the SLM to focus the light on the sample to a diffraction-limited spot. The images of the lateral scattered light were captured at regular intervals (in 2 nm wavelength steps) as the frequency of the incident light was tuned from 6100 cm$^{-1}$ to 10000 cm$^{-1}$. The intensity images were integrated along the height, corresponding to the $y$–axis. This depth-dependent intensity inside the crystal exhibits an exponential decay convolved with a Gaussian instrument response function, which determines the optical resolution.

Since the precise estimation of the resolution of the lateral scattering imaging setup is important in correctly estimating the extinction length, we first turn to this issue. The peak at the input edge of the photonic crystal was modeled with a Gaussian to extract the resolution. Figure S1 shows the Gaussian full-width at half-maximum (FWHM) extracted from the fit. We extract the numerical aperture for the collection arm to be NA = 0.35, which compares well with the nominal collection objective aperture NA = 0.42. It is reasonable that the effective aperture is slightly less than the nominal one, in view of some shadowing by the focusing objective at the input plane (the $xy$–plane).

**FIG. S1.** The estimated Gaussian full width at half-maximum (FWHM) using the lateral-scattered images captured using the camera is shown versus input light frequency. We model the measurements to extract the collection aperture of the setup to be NA = 0.35. The dashed curves are the 95% confidence interval of the model.

**FIG. S2.** The estimated extinction length $\ell_{\text{ext}}$ of TE-polarized light propagating inside the 2D photonic crystal is plotted over a range of frequencies. The frequency-dependence of the extinction length closely follows earlier theoretical predictions of a power-law dependence with an exponent of -2.2. The gray shaded regions correspond to the measured stop gaps. The Bragg lengths $L_B$ as estimated from the photonic strength $S$ are marked as stars at the centers of the stop gaps.

[Graphs showing extinction length versus frequency and spatial resolution measurements]
EXTINCTION LENGTH OF LIGHT IN TWO-DIMENSIONAL PHOTONIC CRYSTALS

Using the data from Fig. S1, we are now in a position to deconvolve the lateral scattered light with the corresponding Gaussian function. We illuminate the crystal with TE-polarized light and capture images of the laterally-scattered light with an analyser crossed with the incident polarization. The cross-polarized measurement configuration enables us to isolate the multiple scattered light as Bragg reflectance from the crystal is sensitive to the polarization [24]. The deconvolved data were matched to a single exponential model to extract the extinction length \( \ell_{\text{ext} \perp} \). The subscript \( \perp \) indicates that we collect laterally-scattered light that is cross-polarized to the incident. In contrast, if we collect only the copolarized laterally-scattered light, the extracted extinction length is dominated by the Bragg length of the crystal. Figure S2 shows the extracted \( \ell_{\text{ext} \perp} \) (circles) as a function of frequency for TE-polarized light incident on the photonic crystal. The error bars correspond to the 95% confidence bound of the fitted extinction length. The extinction length decreases with increasing frequency, as expected, from about \( \ell_{\text{ext} \perp} = 6 \mu m \) at the lowest frequencies in the first order stop band to about \( \ell_{\text{ext} \perp} = 2.5 \mu m \) at the highest frequency beyond the second stop band.

To put these observations in perspective, we compare to theoretical work. Koenderink et al. predicted a power-law dependence of the extinction length on the frequency [36]. For two-dimensional photonic crystals made of infinite long cylinders, the predicted dependence outside the stop gap is \( \ell_{\text{ext}} = A_0 \nu^{-2.2} \), where \( A_0 \) is a scaling parameter that depends on the degree and nature of the disorder. We adjusted only the scaling parameter to the measured data in Fig. S2 and observe that the power-law dependence on frequency agrees very well with the measurements. The observed deviation at low frequencies in the first stop gap is attributed to our choice of limiting the model to a single exponential to describe the intensity inside the crystal. Even in the cross-polarized collection, Bragg interference inside the stop gap leads to additional extinction of light with the Bragg length \( L_B \) as the characteristic length scale, depicted as star markers in Fig. S2. The two length scales arising from multiple scattering and Bragg length would thus require a bi-exponential model to the intensity attenuation within the crystal, which is at this time difficult to significantly model, given the limited dynamic range and signal-to-noise ratio of the data. At frequencies in the range of the second stop gap, the Bragg length is close to the extinction length, and thus it is not sensible to try to fit the data with a bi-exponential model.

WAVE FRONT SHAPING OF LIGHT IN THE PHOTONIC CRYSTALS

The error bars of the enhancements \( E_W \) in Fig. 5 have nearly equal size, independent of the absolute magnitude of the enhancement. We surmise that the enhancement \( E_W \) is Gaussian distributed, typical of independent observations, with a standard deviation given by the observed error bar. Conversely, the enhancement does not match with Poisson statistics, where the error bar would grow with magnitude.

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