



Experimental probe of a complete 3D photonic band gap

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Abstract: The identification of a complete three-dimensional (3D) photonic band gap in real crystals typically employs theoretical or numerical models that invoke idealized crystal structures. Such an approach is prone to false positives (gap wrongly assigned) or false negatives (gap missed). Therefore, we propose a purely experimental probe of the 3D photonic band gap that pertains to any class of photonic crystals. We collect reflectivity spectra with a large aperture on exemplary 3D inverse woodpile structures that consist of two perpendicular nanopore arrays etched in silicon. We observe intense reflectivity peaks ($R > 90\%$) typical of high-quality crystals with broad stopbands. A resulting parametric plot of s-polarized versus p-polarized stopband width is linear (" $y=x$ "), a characteristic of a 3D photonic band gap, as confirmed by simulations. By scanning the focus across the crystal, we track the polarization-resolved stopbands versus the volume fraction of high-index material and obtain many more parametric data to confirm that the high-NA stopband corresponds to the photonic band gap. This practical probe is model-free and provides fast feedback on the advanced nanofabrication needed for 3D photonic crystals and stimulates practical applications of band gaps in 3D silicon nanophotonics and photonic integrated circuits, photovoltaics, cavity QED, and quantum information processing.

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1. Introduction

Completely controlling the emission and the propagation of light simultaneously in all three dimensions (3D) remains a major outstanding target in the field of Nanophotonics [1–5]. Particularly promising tools for this purpose are 3D photonic crystals with spatially periodic variations of the refractive index commensurate with optical wavelengths. The photon dispersion relations inside such crystals are organized in bands, analogous to electron bands in solids [6,7] see, for example, Fig. 1(a). When light waves inside a crystal are Bragg diffracted, directional energy gaps – known as stop gaps – arise for the relevant incident wavevector (see yellow and hatched regions in Fig. 1(a)). When the stop gaps have a common overlap range for all wavevectors and all polarizations, the 3D nanostructure has a photonic band gap as indicated by the pink bar in Fig. 1(a). Within the band gap, no light modes are allowed in the crystal due to multiple Bragg interference [8–10], hence the density of states (DOS) strictly vanishes. Since the local density of states also vanishes in a 3D photonic band gap, the 3D gap is a powerful tool to radically control spontaneous emission and cavity quantum electrodynamics (QED) of embedded quantum emitters [11–14]. Applications of 3D photonic band gap crystals range from dielectric reflectors for antennae [15] and for efficient photovoltaic cells [16–18], via white light-emitting diodes [19], mode and polarization converter [20] to elaborate 3D waveguides [21,22], for 3D photonic integrated circuits [23], to thresholdless miniature lasers [24] and to devices that control

quantum noise for quantum measurement, amplification, and information processing [14,25]. In order to tune the photonic band gap and the stop gaps to frequencies desired for such applications, one may vary the volume fraction of the air (or conversely of the high-index backbone) as is shown in Fig. 1(b).

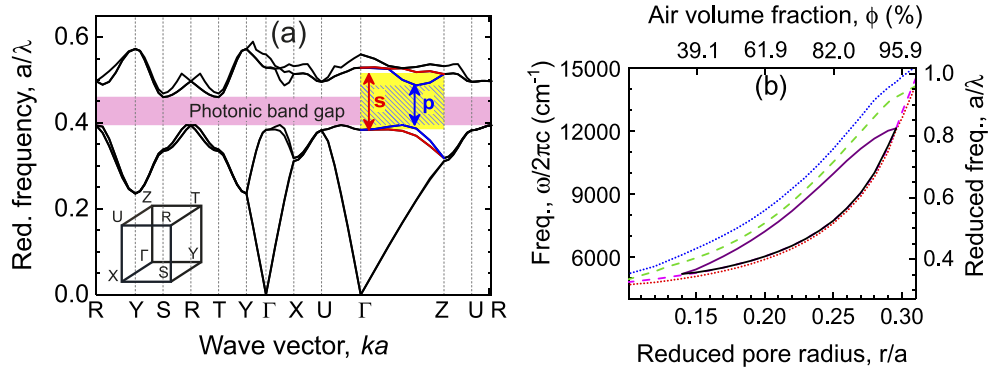


Fig. 1. (a) Band structures calculated for an inverse woodpile photonic crystal for $r/a = 0.19$ and relative permittivity $\epsilon_{Si} = 11.68$. The abscissa is the wave vector in the 1st Brillouin zone (see inset). The experimentally relevant ΓZ high-symmetry direction is enlarged for clarity. The ΓZ stop gaps for s and p -polarized light are indicated by the yellow and hatched bars, respectively. The p -polarized bands are shown in blue and s bands in red [26]. The pink bar is the 3D photonic band gap. (b) The ΓZ stop gaps and 3D photonic band gap as a function of the reduced pore radius r/a , with the corresponding air volume fraction as the top abscissa. The solid curves are the edges of the 3D band gap. The ΓZ stop gap edges are shown as the blue and red dotted curves (p -polarization) and the green and magenta dashed curves (s -polarization). The left ordinate is for a lattice parameter $a = 680$ nm.

Thanks to extensive efforts in nanotechnology, great strides have been made in the fabrication of 3D nanostructures that interact strongly with light such that they possess a 3D complete photonic band gap [14,27–29]. Remarkably, however, it remains a considerable challenge to decide firstly whether a 3D nanostructure has a *bona fide* photonic band gap functionality or not, and secondly to assess how broad such a band gap is, which is critical for the robustness of the functionality. It is natural to try to probe the photonic band gap via its influence on the DOS and LDOS by means of emission spectra or time-resolved emission dynamics of emitters embedded inside the photonic crystal [30–33]. However, such experiments are rather difficult for several practical reasons, that notably involve the emitter’s quantum efficiency [34], the choice of a suitable reference system [35], and finite-size effects [36].

Alternatively, the presence of a gap in the density of states may be probed by transmission or reflectivity [37–52]. In such an experiment, a peak in reflectivity or a trough in transmission identifies a stopband in the real and finite crystal that is interpreted with a directional stop gap in the dispersion relations. By studying the 3D crystal over a sufficiently large solid angle, one expects to see a signature of a 3D photonic band gap. While reflectivity and transmission are readily measured, such probes suffer from two main limitations. One technical impediment is when a reflectivity or transmission experiment samples a too small angular range to safely assign a gap, whereas a broader range would reveal band overlap. The second class of impediments includes possible artifacts related to uncoupled modes [53,54], fabrication imperfections, or unavoidable random disorder, all of which may lead either to erroneously assigned band gaps (‘false positive’) or to overlooked gaps (‘false negative’). To date, these issues are addressed by supplementing reflectivity or transmission experiments with theoretical or numerical results and deciding the presence of a band gap and its width from such results. Theory or numerical

simulations, however, always require a model for the photonic crystal's structure and the building blocks inside the unit cell. Such a model is necessarily an idealization of the real crystal structure and thus misses essential features. For instance, crystal models are often taken to be infinitely extended and thus lack an interface that fundamentally determines reflectivity or transmission features [26]. Or unavoidable disorder is not considered, whereas a certain degree of disorder may completely close a band gap [55]. Or the crystal structure model lacks random stacking (occurring in self-organized structures) which affects the presence and width of a band gap [56]. Thus, when the ideal model differs from the real structure, the optical functionality of the crystal differs from the expected design for reasons that are far from trivial to identify [57]. Therefore, the goal of this paper is to find a purely experimental identification of a photonic band gap that is robust to artifacts as it avoids the need for modeling.

To arrive at a purely experimental probe of the band gap, we exploit the fact that a 3D photonic band gap is a common gap for both polarizations at all wave vectors in the Brillouin zone simultaneously, *cf.*, Fig. 1(a). In an experimental situation, sampling as many wave vectors as possible corresponds to sampling an as large as possible numerical aperture NA, in which case the observed stopband widths for *s* and *p*-polarized light will be equal. Hence, in a parametric plot of the *p*-polarized stopband width versus the *s*-stopband width, the resulting data point is on the straight line ("y = x") through the origin, as illustrated in Fig. 2. Conversely, in the limit of a very small aperture (NA ↓ 0) one samples a gap for only one wave vector, such as the high-symmetry Γ Z stop gap shown in Fig. 2. Since directional stop gaps are polarization sensitive, as is apparent from Fig. 1, in the parametric plot in Fig. 2 the corresponding data clearly deviate from linear behavior. Therefore, the proposed probe of a 3D photonic band gap consists of the following three steps: 1) measure polarization-resolved reflectivity with a high numerical aperture; 2) parametrically plot the widths of the *s* versus the *p*-polarized stopbands; 3) verify how close the

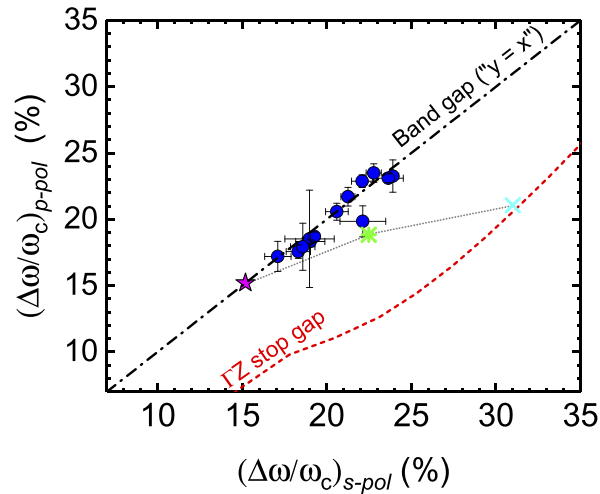


Fig. 2. Parametric plot of relative stopband width for p-polarization versus relative stopband width for s-polarization measured with NA = 0.85 at the same position on crystals with a range of volume fractions (blue circles). Black dashed-dotted line is the linear "y=x" dependence characteristic of the 3D photonic band gap. The red dashed curve pertains to the Γ Z stop gap, as obtained from band structures. The cyan cross and green asterisk are numerical results for normal incidence (NA = 0) and angle-averaged (NA = 0.65) stopbands for $r/a = 0.19$, respectively, and the magenta star is the band gap width simulated for a finite-thickness crystal with $r/a = 0.19$ that are connected by the gray dotted line as a guide to the eye [26].

measured result approaches the band gap limit. In this paper, we experimentally realize such a probe. In addition we add a 4th point, namely, we track the stopband widths versus volume fraction to obtain many parametric data points that all agree with the band gap expectation. In the process, our method is validated by the very good agreement between stop band widths measured as a function of volume fraction and theoretical results for the photonic band gap. Our purely experimental approach is robust and pertains to any crystal structure, including inverse opals and direct woodpiles, as well as aperiodic band gap structures [58], since no *a priori* assumption is made about the sample structure or any other property.

2. Samples and experimental

2.1. Inverse woodpile crystals

Here we study 3D photonic band gap crystals with the inverse woodpile crystal structure [59] made of silicon by CMOS-compatible means. The inverse woodpile structure is designed to consist of two identical two dimensional (2D) arrays of pores with radius R running in the perpendicular X and Z directions. Each 2D array of pores has a centered-rectangular structure with lattice constants a and c in a ratio $a/c = \sqrt{2}$ for the crystal structure to be cubic with a diamond-like symmetry, as illustrated in a YouTube animation [60]. Inverse woodpile crystals have a broad 3D photonic band gap, as shown in Fig. 1, on account of their diamond-like structure [61]. The band gap has a maximum relative bandwidth of 25.4% for a reduced pore radius $r/a = 0.245$ at a relative permittivity $\epsilon_{Si} = 11.68$ typical of silicon as a high-index backbone [62,63].

In our experiments, the axis of the incident light cone is centered on the ΓZ high symmetry direction. Figure 1(a) shows that several bands have s or p -polarized character following the assignment of Devashish *et al.* [26]. This Bloch mode polarization indicates the mode symmetry properties while being excited with either s or p -polarized light incident from a high-symmetry direction (here the Z -direction). Figure 1(a) also shows that the relative bandwidth of the ΓZ stop gap, gauged as the gap width $\Delta\omega$ to mid-gap ω_c ratio, is wider for s -polarized light ($\Delta\omega/\omega_c = 36.5\%$) than for p -polarized light ($\Delta\omega/\omega_c = 27.6\%$), which is reasonable since in the former case the electric field is perpendicular to the first layer of pores so that light scatters more strongly from this layer. For the diamond-like inverse woodpile structure, the ΓZ high-symmetry direction is equivalent to the ΓX high-symmetry direction, and thus also their opposite counterparts *viz.* $-\Gamma Z$ and $-\Gamma X$ [26,49].

Figure 1(b) shows the ΓZ stop gaps for s and p polarization as a function of r/a , as well as the photonic band gap [49]. An increasing pore radius corresponds to an increasing air volume fraction, hence to a decreasing effective refractive index. All gap centers shift to higher frequencies which makes sense, since a gap center frequency ω_c is equal to $\omega_c = \frac{c'}{n_{eff}} \cdot k_{BZ} \cdot G$ [14,64], with c' the speed of light (not to be confused with the lattice parameter c), n_{eff} the effective refractive index of the photonic crystal [65], and G a structure factor [6]. The 3D photonic band gap exists within the broad range $0.14 < r/a < 0.29$ with a maximum width at $r/a = 0.245$, as reported earlier [62,63]. When comparing the stop gaps and the 3D photonic band gap, we note that all lower edges nearly overlap, which is robust as a function of pore radius (r/a), and which is a convenient yet coincidental feature of inverse woodpile crystals that we exploit to validate the volume fraction that we determine by optical means.

The crystals are fabricated by etching pores into crystalline silicon using CMOS-compatible methods [66]. We employed deep reactive ion etching through an etch mask that was fabricated on the edge of a silicon beam [67–69]. Eleven crystals with different design pore radii r_d and a constant lattice parameter $a = 680$ nm were fabricated on the silicon beam. Figure 3(a) shows a scanning electron microscopy (SEM) image of one of our crystals with designed pore radius $r_d = 160$ nm ($r_d/a = 0.235$). The dimensions of each crystal are typically $8 \times 10 \times 8 \mu\text{m}^3$.

Figure 3(a) shows that the sample geometry allows for good optical access to the XY and YZ crystal surfaces.

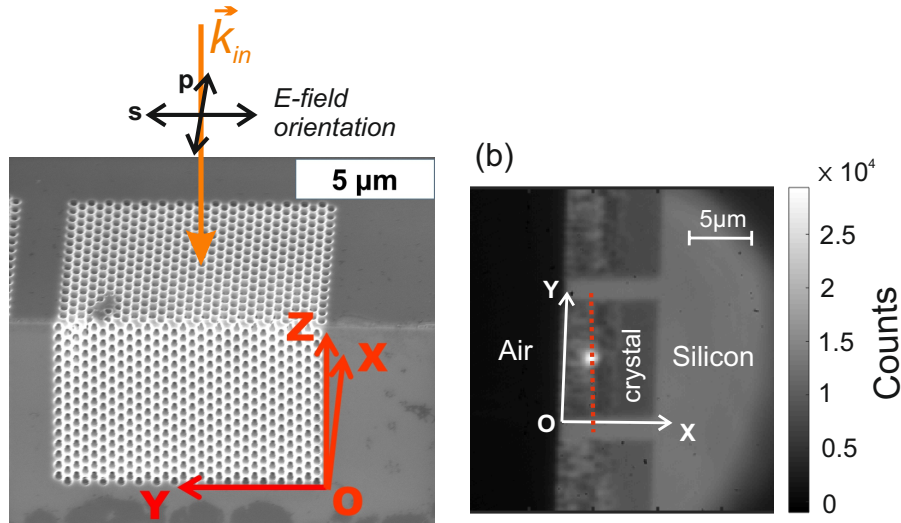


Fig. 3. (a) Scanning electron microscopy (SEM) image of the edge of the silicon beam with a cubic 3D inverse woodpile crystal in perspective view. The crystal consists of two sets of perpendicular pores along the X and Z directions with design radius $r_d = 160\ \text{nm}$. The coordinate system used in the paper is shown with the origin at the lower right corner of the crystal. The crystal has lattice parameters $a = 680\ \text{nm}$ in the Y -direction, and c in the X and Z -directions with $c = a/\sqrt{2}$. Top: The incident light cone is centered around the wave vector $\vec{k}_{in}$ in the ΓZ direction. The polarization is shown: light is p -polarized when the incident E-field is parallel to the X -directed pores, and s -polarized when the incident E-field is perpendicular to the X -directed pores. (b) Image of the XY -surface of one of the 3D inverse woodpile crystals taken with the IR camera in the optical setup with near infrared LED illumination, with two partly visible neighboring crystals below and above. The bright spot on the crystal is the focus of the incident light from the supercontinuum source filtered by the monochromator. The dotted red line shows the position scan of the focus across the crystal as shown in Fig. 6.

2.2. Near-infrared reflectivity microscope

We have developed a near-infrared microscope setup to collect position-resolved broadband reflectivity spectra of photonic nanostructures, as is shown in Fig. 4. The near-infrared range of operation is compatible with 3D silicon nanophotonics as it avoids the intrinsic silicon absorption. The setup was developed with the option to collect in future light scattered perpendicular to the incident light. Furthermore, a spatial light modulator can be inserted to eventually perform wavefront shaping [70–72]. Therefore, we decided to use sequential scanning of wavelengths instead of measuring the spectrum at once with a spectrometer as in [38,49,73].

In the optical setup shown in Fig. 4, the silicon beam with the 3D crystals is mounted on an XYZ translation stage that has a step size of about $30\ \text{nm}$. We use a broadband supercontinuum source (Fianium SC 450-4, $450\ \text{nm} - 2400\ \text{nm}$) whose output is filtered by a long pass glass filter (Schott RG850) to block the unused visible range. The near infrared light is spectrally selected by a monochromator (Oriel MS257; $1200\ \text{lines/mm}$ grating) with an output linewidth of about $\Delta\lambda = 1\ \text{nm}$ and a tuning precision better than $0.2\ \text{nm}$. The accessible range of wavelengths spans from $900\ \text{nm}$ to $2120\ \text{nm}$ (or wave numbers $\omega/2\pi c = 11000\ \text{cm}^{-1}$ to $4700\ \text{cm}^{-1}$) that

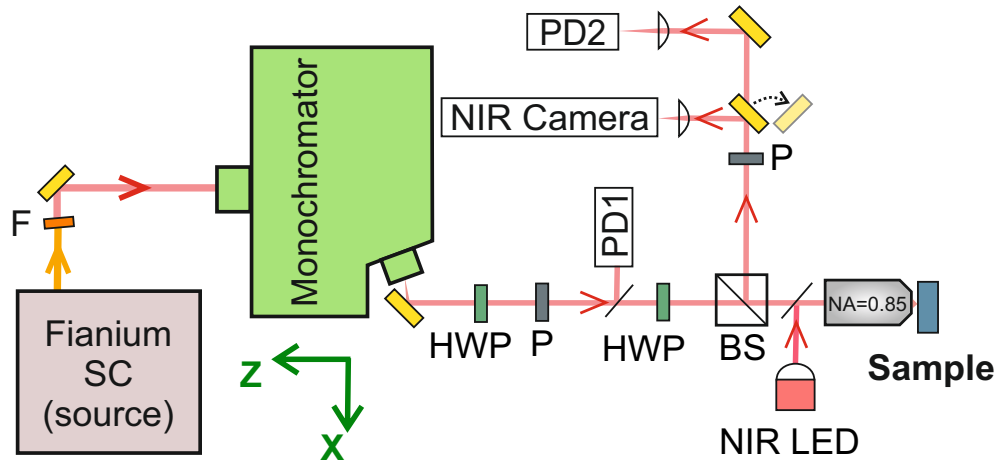


Fig. 4. Setup to measure position-resolved microscopic broadband reflectivity. The Fianium SC is the broadband supercontinuum source, the long-pass glass filter F blocks the visible light at $\lambda < 850$ nm, the monochromator filters the light to a narrow band, HWP are half-wave plates, P are polarizers, and BS are beam splitters. Incident light is focused on the sample with a $100\times$ objective that also collects the reflected light; the coordinate system is shown at top right. The NIR camera views the sample in reflection with an effective magnification of $250\times$. The photodiodes PD1 and PD2 monitor the incident light power and measure signal from the crystal, respectively.

includes the telecom bands. Using a combination of a linear polarizer and half wave plates, the linear polarization of the spectrally filtered light is selected and sent to an infrared apochromatic objective (Olympus LC Plan N $100\times$) to focus the light onto the sample's XY surface with a numerical aperture $NA = 0.85$. The NA corresponds to a collection solid angle of 0.95π sr. On account of the crystal symmetry mentioned above (ΓZ equivalent to ΓX , $-\Gamma Z$ and $-\Gamma X$), we effectively collect a solid angle of 3.8π sr.

Light reflected by the sample is collected by the same objective as shown in Fig. 4. A beam splitter directs the reflected light towards the detection arm where the reflection from the sample is imaged onto an IR camera (Photonic Science InGaAs). In order to locate the focus of the input light on the surface, a near infrared LED is used to illuminate the sample surface. We use the XYZ translation stage to move the sample to focus the light on the desired location. An image as seen on the IR camera (see Fig. 3(b)) reveals the XY surface of the Si beam. The bright circular spot with a diameter of about $2\ \mu\text{m}$ is the focus of light reflected from the crystal. The rectangular darker areas of about $8\ \mu\text{m} \times 10\ \mu\text{m}$ are the XY surfaces of the 3D photonic crystals. They appear dark compared to the surrounding silicon since the LED illumination is outside the band gap of these crystals whose effective refractive index is less than that of silicon.

Once the input light beam is focused on the sample, the reflected light is sent to photodiode PD2 (Thorlabs InGaAs DET10D/M, 900 nm - 2600 nm) by flipping off the mirror in front of the camera. The photodiode records the reflected intensity I_R as the monochromator scans the selected wavelength range. An analyzer in front of the detector selects the polarization of the reflected light. All reflectivity measurements are done for two orthogonal polarization states of the incident light, namely s (electric field transverse to X-directed pores) and p (electric field parallel to X-directed pores), as defined in Fig. 3(a). A typical spectrum takes about 5 to 25 minutes to record depending on the chosen wavelength step size (typically 10 nm or 2 nm). Using

the translation stage, the sample is moved in the Y-direction to select different crystals on the edge of the silicon beam.

To calibrate the reflectivity defined as $R \equiv I_R/I_0$, the spectral response I_R of the crystals is referenced to the signal I_0 from a clean gold mirror that reflects 96%. Referencing also removes dispersive contributions from optical components in the setup. To ensure that the signal to noise ratio of the photodiode response is sufficient to detect signal in the desired range, the detector photodiode is fed into a lock-in amplifier to amplify the signal with a suitable gain. Since a serial measurement mode holds the risk of possible temporal variations in the supercontinuum source, we simultaneously collect the output of the monochromator with photodiode PD1 in each reflectivity scan. This monitor spectrum is used to normalize variations in the incident intensity I_0 . Since it is tedious to dismount and realign the sample to take reference spectra during a position scan, we also take secondary reference measurements on bulk silicon outside the crystals, which has a flat response $R \approx 31\%$ with respect to the gold mirror.

To verify the reproducibility of our experiments (both the fabrication methods and the optical measurements), we include in this paper data obtained with an older setup on an older silicon bar. Since several crystals on this bar have been characterized by traceless X-ray tomography [57], the results on these crystals validate the optical method described below to determine the pore size.

3. Results

3.1. Reflectivity and stopband

Figure 5 shows reflectivity spectra measured on three 3D crystals with different designed pore radii $r_d = 130, 140, 160$ nm, as well as on the Si substrate. The constant reflectivity $R = 30.6 \pm 1.3\%$ of the substrate agrees well with the Fresnel reflectivity of 31% expected for bulk silicon at normal incidence [74]. Intense reflectivity peaks with maxima of $R_m = 96\%$ and 94% are measured on the crystals with pore radii $r_d = 130$ nm and 140 nm, respectively. A slightly lower maximum reflectivity of 70% observed for the $r_d = 160$ nm crystal is caused by the Si etching process that seems to produce smoother pores at smaller radii. Our observations are consistent with recent numerical results that perfect silicon inverse woodpile crystals with a thickness of only three unit cells reflect 99% of the incident light [26]. Our results are also consistent with 95% reflectivity measured by Euser *et al.* on a direct silicon woodpile that was only one unit cell thick [75]. We surmise that the current maximum reflectivities are higher than those of [49,68] due to improved nanofabrication and improved optics.

The reflectivity peaks correspond to the stopband and are associated with the main ΓZ stop gap centered near $a/\lambda = 0.45$ in Fig. 1(a). Figure 5 also shows that the center of the stopband shifts to higher frequencies with increasing pore radius, which qualitatively agrees with the calculated behavior shown in Fig. 1(b).

The stopband width is taken as the full width at half maximum (FWHM) of the reflectivity peak [76]. The baseline of the peak is taken as the minimum reflectivity in the long-wavelength limit at frequencies below the stopband, with the standard deviation in this frequency range as the error margin. Similarly, the maximum reflectivity is taken as the mean in a narrow frequency range around the peak, with the standard deviation in this range taken as the error margin. The errors are propagated into the estimates of the edges at half maximum of the peak.

3.2. Position-dependent stopband

It is well-known from structural studies such as scanning electron microscopy on cleaved crystals [66] and from non-destructive X-ray tomography [57] that the radius of etched nanopores varies slightly around the designed value with depth inside the crystal due to the nature of the etching process [66]. By comparing the lower edge of the measured stopband with the calculated stop gap (*cf.* Fig. 1(b)), we obtain an estimate of the local average pore radius r at the position (X, Y, Z) of

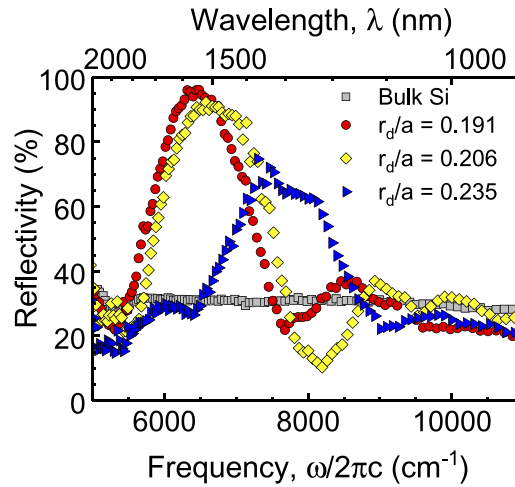


Fig. 5. Reflectivity spectra of three different 3D photonic crystals with three designed pore radii $r_d = 130, 140$ and 160 nm ($r_d/a = 0.191, 0.206, 0.235$) (red circles, yellow diamonds and blue triangles, respectively). The stopbands appear at different frequency ranges. The gray squares represent reflectivity from bulk Si on the beam away from the crystals.

the optical focus: $r(X, Y, Z)$. In this comparison we profit from the feature in the band structures of inverse woodpile crystals that the lower edges of both the band gap and of the stop gap are nearly the same, hence the determination is robust to the interpretation which gap is probed.

For the three spectra in Fig. 5, we derive the pore radii to be $r/a = 0.190 \pm 0.001$, 0.195 ± 0.001 , and 0.228 ± 0.002 , respectively, which agrees very well with the design ($r_d/a = 0.191, 0.206, 0.235$), where the small differences are attributed to the depth-dependent pore radius discussed above. We note that since the probing direction is perpendicular to the X -directed pores in the crystals, the derived pore radii are effectively those of the pores that run in the X -direction.

Next, we collect reflectivity spectra while scanning the focus across the crystal surface. Since we then effectively scan the pore radius r , we expect to scan the stopband in response. As an example, Fig. 6 shows the results of a Y -scan across one of our crystals with design pore radius $r_d = 130$ nm ($r_d/a = 0.191$). The position scan of the focus across the crystal is shown as the red dashed line in the camera image shown in Fig. 3(b). While scanning the Y -position, a slight excursion occurred in the X -direction from $X = 2.8 \mu\text{m}$ to $3.2 \mu\text{m}$ due to imperfect alignment of the silicon beam axis with the vertical axis of the translation stage. From each collected spectrum, we derive the peak reflectivity R_m and the minimum reflectivity below the stopband R_l as shown in Fig. 6(a). Inside the crystal there is substantial difference between R_m (up to $R_m = 94.8\%$) and R_l , hence the crystal's reflectivity peaks are well-developed. Near the crystal edges ($Y = 0 \mu\text{m}$ and $10 \mu\text{m}$) the difference between R_m and R_l rapidly decreases and both tend to about 31% since the focused light here is reflected by bulk silicon.

Figure 6(b) shows the edges of the measured stopband as a function of Y . Between $Y = 0 \mu\text{m}$ and $10 \mu\text{m}$ the lower edge shifts down from 5950 to 5550 cm^{-1} and the upper edge shifts down from 7550 to 6550 cm^{-1} . In other words, both the center frequency of the stopband and its width decrease with increasing Y as a result of the variation of the pore radii with position. The redshift of the stopband frequencies is mostly caused by the small excursion along X , since the radius of the X -directed pores decreases with increasing X .

By comparing the measured lower edges in Fig. 6(b) with the theoretical gap maps shown in Fig. 1(b), we derive the local pore radius $r(X, Y, Z)$ in the crystal that is plotted versus Y -position

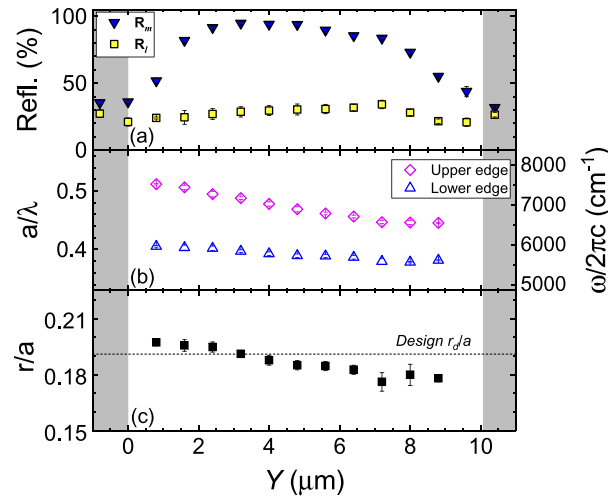


Fig. 6. Reflectivity measured as a function of Y -position on a crystal with design pore radius $r_d = 130$ nm (or $r_d/a = 0.191$), measured with p -polarized light. (a) Maximum peak reflectivity (R_m) and minimum reflectivity below the stopband (R_l). (b) Upper edges (magenta diamonds) and lower edges (blue triangles) of the stopband obtained from the half heights of the reflectivity peaks. The right ordinate is absolute frequency for a lattice parameter $a = 680$ nm. (c) Relative radii r/a derived by comparing the lower edge of the stopband with data shown in Fig. 1(b). The grey areas at $Y < 0$ μm and $Y > 10$ μm indicate bulk silicon outside the crystal with a constant reflectivity near 31%.

in Fig. 6(c). The resulting $r(X, Y, Z)/a$ is seen to vary from 0.197 to 0.176 about the design pore radius $r_d/a = 0.191$. Therefore, we can now combine all position-dependent data to make maps of stopband centers and stopband widths as a function of the pore radius.

3.3. Gap map from experiments

We have applied the procedures described in sections 3.2 and 3.1 to reflectivity measured on many crystals and we also collected spectra during Y -scans on two crystals to verify the consistency of all observations. From all collected reflectivity spectra, both s and p polarized, the lower and upper stopband edges are extracted, and are mapped as a function of r/a in Fig. 7. The lower edge data form a continuous trace from reduced frequency $a/\lambda = 0.38$ at $r/a = 0.17$ to $a/\lambda = 0.50$ at $r/a = 0.245$. The data match well with the theory, which is obvious since we used the lower edge to estimate r/a from the measured spectra.

The upper edge data form a continuous trace from reduced frequency $a/\lambda = 0.42$ at $r/a = 0.17$ to $a/\lambda = 0.64$ at $r/a = 0.245$. It is remarkable that the upper edge data for both s and p -polarized light mutually agree very well, especially for pore radii $r/a > 0.21$. This observation implies that the measured stopband is representative of the 3D photonic band gap that is polarization insensitive, as opposed to a directional stop gap that is polarization sensitive.

In comparison to theory, at pore radii $r/a < 0.21$ the upper edges are in between the theoretical upper edges of the band gap and the p -polarized edge of the directional stop gap. At larger radii ($r/a > 0.21$), all measured upper edge data are near the theoretical upper band gap edge and differ from the stopband edges. This observation adds support to the notion that the structure-dependent stopbands represent the 3D photonic band gap, rather than a directional stop gap.

We plot in Figs. 8(a) and 8(b) the relative stopband width (gap to mid-gap ratio) as a function of r/a as derived from the lower edges. The large number of data in Fig. 8(a) shows that the width of the s -polarized stopband increases up to $r/a = 0.2$ before saturating up to $r/a = 0.24$. The

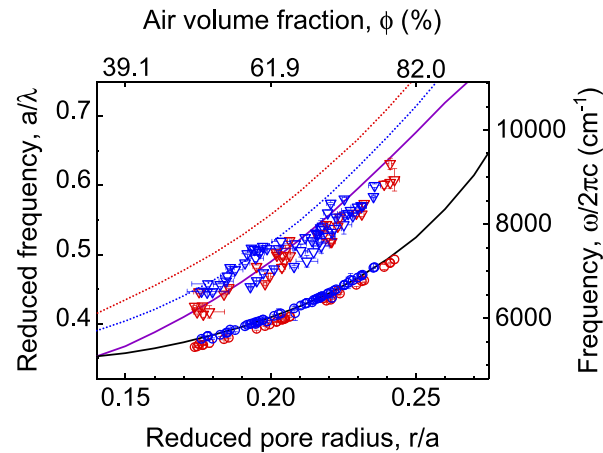


Fig. 7. Evolution of the stopband edges versus pore radius. The red and blue triangles represent upper edge of the stopband for *s* and *p*-polarized light respectively. The red and blue circles represent the lower edge of the stopband for *s* and *p* polarized light. The stopband edges are inferred from the reflectivity peak measured on 11 crystals. The solid lines indicate the edges of the photonic band gap. The upper edge of the ΓZ stop gap for *s* and *p* polarized light are plotted as the red and blue dotted curves, respectively. The right ordinate is absolute frequency for a lattice parameter $a = 680$ nm.

s-polarized data for an older Si beam agree well with our data, except for an outlier at $r/a = 0.24$. For these older crystals, the pore size r/a was obtained from a direct structure-determining method, namely X-ray tomography [57]. Consequently, the good agreement with the newer crystals whose pore radii are determined from the stop band edge validates the optical determination of the pore radii. Moreover, since the optical experiments on the older crystals employed a different setup, the good agreement indicates that both the old and the new reflectivity spectra are representative, even though the old setup yields lower maximum reflectivities. All data are close to the theoretical prediction for the width of the 3D photonic band gap and lie distinctly below the theoretical width of the stop gap.

Figure 8(a) also shows results of *s*-polarized reflectivity simulated for a *finite* inverse woodpile crystal with $r/a = 0.19$ [26], namely of a directional stopband, of an angle-averaged stopband (for a range of angles relevant for a reflecting objective with $NA = 0.65$), and of an omnidirectional band gap. With increasing aperture, the simulated stopband becomes narrower. From the comparison, it is apparent that our data match best with the width of the 3D photonic band gap.

Figure 8(b) shows the *p*-polarized stopband widths versus pore radius. At pore radii $r/a < 0.21$, the stopband widths are in between the theoretical bandwidths of either the directional stopgap or the omnidirectional band gap. At larger radii ($r/a > 0.21$), the measured stopband widths match better with the theoretical width of the band gap than with the stop gap width. From *p*-polarized finite-crystal simulations done at $r/a = 0.19$ [26], we conclude that the bandwidths of the directional stop gap, of the angle-averaged stopgap, and of the band gap are near to each other, hence it is difficult given the variations in our data to discriminate between either feature. Considering the *s* and *p*-polarized stopband widths jointly, we again find a much better agreement with the 3D photonic band gap than with the directional stop gap.

The conclusions from Figs. 7 and 8 are based on the agreement between measurements on one hand, and simulations and theory on the other hand. The latter invokes an idealized structural model, for instance, pores as infinite perfect cylinders which neglect pore tapering, or roughness. Therefore, these conclusions do not represent a purely experimental probe of a 3D band gap.

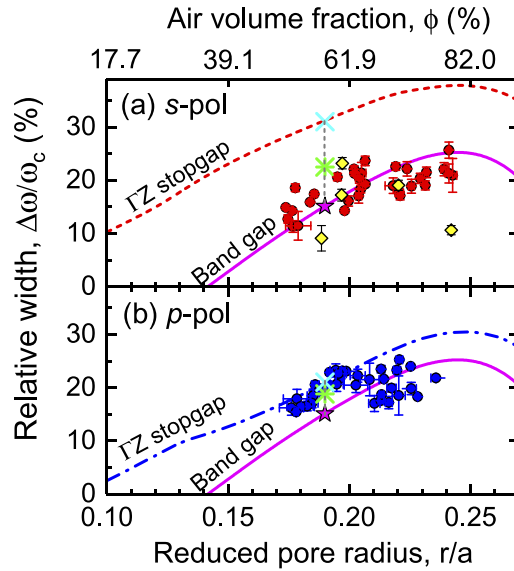


Fig. 8. Measured relative stopband width (gap width to midgap, $\Delta\omega/\omega_c$) versus r/a for (a) s -polarized (red circles), (b) p -polarized (blue circles) input light. Yellow diamonds in (a) are data from an older Si beam. The cyan crosses, green asterisks, and magenta stars are numerical results for normal incidence, angle-averaged stopband, and complete band gap at $r/a = 0.19$, respectively for both polarizations [26]. The dashed red and dash dotted blue curves represent the width of the ΓZ stop gap obtained from band structures for s and p polarized light, respectively. The magenta solid curve is the 3D photonic band gap from band structures.

3.4. Experimental probe of the photonic band gap

At this point, we are in a position to complete the model-free experimental probe of a 3D photonic band gap, consisting of the (3+1) step plan outlined in section 1. Up to here, we have discussed the collection polarization-resolved reflectivity spectra using a large NA (step #1). Next, we parametrically plot the width of the measured p -polarized stopband versus the width of the s -polarized stopband that is shown in Fig. 2 (step #2). In order to avoid systematic errors due to the position-dependence of the stopbands, we select data where spectra were measured for both polarizations on the same position on a crystal.

Figure 2 shows that for s -polarized stopband widths between $\Delta\omega/\omega_c = 17\%$ and 24% , the corresponding p -polarized stopband width increases linearly, and also from 17% to 24% . Such a strictly linear increase agrees with the expectations for a 3D photonic band gap even without modeling, since a 3D band gap entails a forbidden gap for *both polarizations simultaneously* [2] (steps #3 and #4). In the case of the alternative hypothesis that the measured stopbands correspond to directional ΓZ stop gaps, the parametric trend would be nonlinear and clearly differ from the diagonal. Since this trend obviously does not match with our data, we reject this hypothesis.

In order to validate our proposed method, we discuss results obtained from numerical simulations on a finite-size inverse woodpile photonic crystal by Devashish *et al.* [26]. The simulations were done for inverse woodpiles made from silicon with a pore radius $r/a = 0.19$, and the incidence angle was varied over a wide range. Several situations were simulated, namely single-direction incidence from a high-symmetry direction (ΓX or ΓZ) with effectively zero numerical aperture ($NA = 0$). Secondly, simulations were done for incidence over a large range

of angles corresponding to a reflecting objective with $NA = 0.65$ (see [49,73]). Thirdly, the 3D photonic band gap was studied. The simulations reveal that when NA is increased, the corresponding data move towards the diagonal: Fig. 2 shows that the data point for the directional stop gap agrees very well with the stop gap curve and is far from the diagonal band gap line. The data point simulated for the $NA = 0.65$ objective is in between the stop gap and the band gap curves, as expected since these curves effectively represent low and high NA . Finally, the data point for the band gap agrees well with the band gap prediction and not at all with the stop gap curve. Therefore, the numerical aperture $NA = 0.85$ used here and the correspondingly large overall solid angle of 3.8π sr is apparently sufficient to probe the omnidirectional photonic band gap.

4. Discussion

It is widely agreed that the fabrication of 3D nanostructures necessary for photonic band gap physics is challenging [39–41,77]. Consequently, since the detailed 3D nanostructure critically determines the band gap functionality, it is important to have a non-destructive verification of the functionality. We propose that the practical band gap probe method presented here fills a gap by providing relatively fast feedback on a newly fabricated band gap material. In a holistic approach, one would not only verify the functionality but also the 3D band gap material since the latter usually aids the understanding of the functionality, especially in complex situations where the function differs from the designed one. While studying the detailed 3D structure of a nanostructure is non-trivial, successful methods have been reported using X-ray techniques, notably small-angle X-ray scattering [78–80], X-ray ptychography [81], or traceless X-ray tomography [57].

The optical analysis discussed in section 3.2 provides a relatively straightforward and non-destructive way to study details of the 3D band gap material, whereas in section 3.4 we present a purely experimental probe of the 3D photonic band gap without the need to idealize the crystals as is traditionally done in numerical simulations. Since this experimental probe is independent of the crystal structure, it is readily applicable to other types of 3D photonic band gap materials such as inverse opals, direct woodpiles, and even to non-periodic materials [23,39,58].

So far, the optical analysis discussed in section 3.2 was specific to the inverse woodpile structure studied here [59]. In order to generalize our analysis to other classes of photonic band gap crystals, such as inverse opals, direct woodpiles, and even non-periodic ones [58], it is useful to realize that a varying pore size in an inverse woodpile structure corresponds to the tuning of the filling fraction and thus of the effective refractive index [65], both of which pertain to all other classes of photonic band gap structures. Both the filling fraction and the effective index are readily generalized to other 3D photonic band gap crystals. For instance, in inverse opals the filling fraction of the high-index backbone is known to vary with preparation conditions [79], hence this can be used as a tuning knob. In direct woodpile crystals, the filling fraction is notably tuned by varying the width of the high-index nanorods [23,39], and similarly in hyperuniform structures [58]. It is therefore that the top abscissae in Figs. 7, 8, and 1(b) have been generalized to the effective refractive index. Therefore, the stopband width versus the effective index (as in Fig. 8) or the p -polarized stopband width versus the s -polarized one also pertain as probes to other classes of band gap structures, and thus serve as experimental probes of the 3D photonic band gap in such other structures.

We foresee that a practical probe of 3D photonic band gaps will boost their applications in several innovative fields. For instance, recent efforts by the Tokyo and Kyoto teams have demonstrated the use of 3D photonic band gap crystals as platforms for 3D photonic integrated circuits [23,82]. In the field of photovoltaics that is of considerable societal interest, the use of 3D photonic band gap crystals is increasingly studied to enhance the collection efficiency by means of various kinds of photon management [16,17,83]. A robust band gap is necessary to

realize embedded point or line defects in a 3D photonic crystal to effectively control emission and 3D waveguiding applications [22,43]. It is an essential feature of a 3D photonic band gap crystal to have a gap in the density of states, which in turn corresponds to the density of vacuum fluctuations. Therefore, quantum devices embedded inside a 3D band gap crystal are effectively shielded from quantum noise [25], including quantum gates that manipulate qubits for quantum information processing.

5. Conclusion

In this paper, we present a purely experimental probe of the 3D band gap in real three-dimensional (3D) photonic crystals, without the need for theoretical or numerical modeling that invokes idealized and even infinite photonic crystals. As an exemplary structure, we study 3D inverse woodpile crystals made from silicon. For the probe, we exploit the fact that a 3D photonic band gap is a common gap for both polarizations at all wave vectors in the Brillouin zone simultaneously. The band-gap probe consists of three main steps: 1) measure polarization-resolved reflectivity with a high numerical aperture; 2) parametrically plot the widths of the s versus the p -polarized stopbands; 3) verify how close the measured result approaches the band gap limit. In addition, a 4th point describes how to track the stopband widths versus volume fraction to obtain many parametric data points that all agree with the band gap expectation.

In an experimental situation, sampling as many wave vectors as possible corresponds to sampling an as large as possible numerical aperture NA, in which case the observed stopband widths for s and p -polarized light will be equal. Hence, in a parametric plot of the p -polarized stopband width versus the s -stopband width, the resulting data point is on the straight line (" $y = x$ ") through the origin.

In the process, we have collected position and polarization-resolved reflectivity spectra of multiple crystals with different design parameters with a large numerical aperture and observed intense reflectivity peaks with maxima exceeding 90% corresponding to broad (up to 24%) stopbands, typical of high-quality crystals. We have produced a gapmap for the experimental stopband width versus pore radius, which agrees much better with the predicted 3D photonic band gap than with a directional stop gap. From a parametric plot of s -polarized versus p -polarized stopband width, we obtain a strictly linear dependence, in agreement with the 3D band gap and at variance with the directional stop gap. This parametric plot is a purely experimental probe of the 3D band gap and can be readily applied to other types of 3D photonic band gap crystals. Such a practical probe provides a fast evaluation of the advanced nanofabrication required for 3D photonic crystals. Moreover, the fast probe of 3D band gaps will stimulate practical applications of band gaps, notably in 3D silicon nanophotonics and photonic integrated circuits, photovoltaics, cavity QED, and quantum information processing.

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