Mode density inside an omnidirectional mirror is heavily directional but not small

Christa Hooijer and Daan Lenstra

Division of Physics and Astronomy, Vrije Universiteit Amsterdam, De Boelelaan, 1081, 1081 HV Amsterdam

Ad Lagendijk

Van der Waals–Zeeman Institute, University of Amsterdam, Valckeniersstraat 65-67, 1018 XE Amsterdam

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We show that ominidirectional reflection is not a sufficient signature of a photonic bandgap. Although dramatic angular redistribution takes place, the mode density of the electromagnetic field is hardly altered within the ominidirectional reflection range but rather has characteristics typical of a waveguide. The strikingly large polarization anisotropy is due to the huge dielectric contrast but not to a photonic bandgap.

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Photonic crystals are defined as structures with a dielectric constant that varies periodically in three dimensions on the scale of optical wavelengths.[1] Structures with one-dimensional (1D) or two-dimensional periodic variation of the dielectric constant also exhibit extremely interesting behavior, since because of this periodic variation stop bands for the electromagnetic (EM) field can occur for a certain frequency range, and depending on the dimensionality of the crystal these bands can be either 1D, two dimensional, or three dimensional.[2] When for any direction in three dimensions a frequency range can be identified for which the stop bands overlap, this range is called a photonic bandgap (PBG). From the outside, for frequencies within the stop band, these crystals may exhibit total reflectivity for all angles of incidence in the direction(s) of the stop band. This phenomenon is known as omnidirectional reflection (ODR). This property can be used to make ultralow-loss mirrors and waveguides. Inside the crystal and for frequencies within the stop band, the electromagnetic- (EM-)field mode density is strongly altered, and this alteration may allow inhibited spontaneous emission. For the latter property it is vital that the PBG be three dimensional so that propagation in any direction is impossible. A three-dimensional PBG can be realized only in a three-dimensional photonic crystal. Because of their attractive outside and inside features, photonic crystals are a topic of intense research.[3] Note that for these features to be observed it is not sufficient to locally engineer the EM-field mode density to be zero, since for a PBG the EM-field mode density would be zero everywhere in the crystal.

Recently, a 1D photonic crystal that exhibited spectacular ODR was proposed and demonstrated by Joannopoulos *et al.* at the Massachusetts Institute of Technology.[4] It attracted quite a lot of attention and has many exciting applications. This omnidirectional mirror is a special case of a Bragg mirror with an unusual high contrast between the layers, which are made from tellurium (Te; $n_1 = 4.6$) and polystyrene (PS; $n_2 = 1.6$). Using these parameters, Joannopoulas *et al.* calculated the band structure of an infinite 1D crystal, identifying regions in the (ω, k) plane in which no propagation is possible (ω is the frequency of light, and k is the wave vector in the directions perpendicular to the variation in the dielectric constant). A very interesting question is what happens to the EM-field mode density inside such a mirror. Although a calculation of the spontaneous-emission rate of an atom embedded in a defect layer sandwiched between two semi-infinite 1D omnidirectional mirrors has been done,[5] surprisingly, it seems that the mode density in such a layer has never explicitly been calculated for a structure that is finite in the direction of periodicity.[6] Nevertheless, since the structure discussed here is essentially 1D, this is an excellent opportunity to calculate the mode density, as the calculation is feasible though far from trivial.

In this Letter we calculate the mode density inside the omnidirectional mirror in a rigorous way to look for the inside signature of the stop band at arbitrary positions inside a finite crystal with nine layers, alternately Te (labeled 1) and PS (labeled 2), ending with a layer of Te, with air on the outside (labeled a), and with all parameters as specified in Ref. 4. The mode density at a certain position $\mathbf{r}$ is given by $\mathrm{Im}[G(\mathbf{r}, \mathbf{r}, \omega)]$, where $G(\mathbf{r}, \mathbf{r}', \omega)$ is the EM Green function specific to a certain structure. One should realize that, in general, owing to the vectorial nature of the EM field, one has a Green tensor. In special cases, including the present configuration, one can split this tensor into two separate Green functions, one for each polarization type. Here we distinguish between TE and TM polarization. The Green functions of a planar multilayer structure with n layers were derived by Tomaš.[7] Since the variation in dielectric constant is in only the z

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direction, it is natural to write $\mathbf{r} = (\boldsymbol{\rho}, z)$. Because of isotropy in the $\boldsymbol{\rho}$ plane, G depends only $(\boldsymbol{\rho} - \boldsymbol{\rho}')$, and thus to calculate $\mathrm{Im}[G(\mathbf{r}, \mathbf{r})]$ we only need to know $\mathrm{Im}[G(z, \omega)]$, which is given for a certain position z_0 in layer j by

$$\mathrm{Im}[G^{\mathrm{TE}}(z_0, \omega)] = \frac{1}{4\pi} \mathrm{Re} \int_0^\infty \mathrm{d}k \, \frac{k}{\beta_j D^{\mathrm{TE}}} \{1 + r_{j+}^{\mathrm{TE}} \times \exp[2i\beta_j(d_j - z_0)]\}[1 + r_{j-}^{\mathrm{TE}} \exp(2i\beta_j z_0)] \quad (1)$$

for TE polarization [$\mathbf{E}$ field in the $(\hat{\mathbf{k}}, \hat{\mathbf{z}})$ plane] and

$$\mathrm{Im}[G^{\mathrm{TM}}(z_0, \omega)] = \frac{1}{4\pi} \mathrm{Re} \int_0^\infty \mathrm{d}k \left(\frac{k^3}{k_j^2 \beta_j D^{\mathrm{TM}}} \{1 + r_{j+}^{\mathrm{TM}} \times \exp[2i\beta_j(d_j - z_0)]\}[1 + r_{j-}^{\mathrm{TM}} \exp(2i\beta_j z_0)] + \frac{k\beta_j}{k_j^2 D^{\mathrm{TM}}} \times \{r_{j+}^{\mathrm{TM}} \exp[2i\beta_j(d_j - z_0)] - 1\}[r_{j-}^{\mathrm{TM}} \exp(2i\beta_j z_0) - 1] \right) \quad (2)$$

for TM polarization ($\mathbf{E}$ field in the $\hat{\mathbf{k}} \times \hat{\mathbf{z}}$ direction); $\mathbf{k} = k\hat{\mathbf{k}}$ is the wave vector in the ρ plane, which is isotropic. Therefore k is equal in all layers, $\hat{\mathbf{k}}$ is a unit vector, $\hat{\mathbf{z}}$ is the unit vector in the z direction, c is the speed of light in vacuum, $\beta_j = [(n_j^2\omega^2/c^2) - k^2]^{1/2}$ is the wave vector in the jth layer with thickness d_j in the z direction, $r_{j\pm}^q$ are the reflection coefficients, and $D^q = 1 - r_{j+}^q r_{j-}^q \exp 2i\beta_j d_j$ is the term that accounts for the multiple reflections at the interfaces.

Taking z_0 at the middle of the middle (Te) layer of this system, we find that the reflection coefficients r_{j+}^q and r_{j-}^q are equal for symmetry reasons and are given by[8]

$$r_{j\pm}^q = \frac{(\mathcal{M}_{11} + \mathcal{M}_{12} p_a^q) p_1^q - (\mathcal{M}_{21} + \mathcal{M}_{22} p_a^q)}{(\mathcal{M}_{11} + \mathcal{M}_{12} p_a^q) p_1^q + (\mathcal{M}_{21} + \mathcal{M}_{22} p_a^q)}, \quad (3)$$

with $p_i^{\mathrm{TE}} = \beta_i$ and $p_i^{\mathrm{TM}} = \beta_i / n_i$. The $\mathcal{M}_{ij}$ are elements of the 2×2 matrix that is characteristic of the two-cell system,[8] in which each cell consists of one PS and one Te layer.

Different regions can be identified for the integrand in Eq. (1). For k values smaller than $n_a\omega/c$ the integrand is continuous; this corresponds to radiation modes that couple light out of the crystal. For k values from $n_a\omega/c$ to $n_1\omega/c$ the absolute value of the reflection coefficients is 1. The real part of the integrand is zero, apart from some well-defined discrete k values, where $D^q = 0$ and a contribution from the pole is found. These k values correspond to the guided modes, i.e., modes that are propagating only inside the crystal in the $\boldsymbol{\rho}$ plane. For k values beyond $n_1\omega/c$ the real part of the integrand is zero everywhere: No propagating solutions exist within the crystal.

We have calculated the integral (1), with $r_{j\pm}^q$ substituted from Eq. (2) and z_0 as indicated above, as a function of ω, making a distinction between the contributions of the radiation modes (continuous part) and the guided modes (discrete part). The results, normalized to the vacuum mode density of the relevant polarization [this means that a factor $\omega/(4\pi c)$ cancels out], are displayed in Fig. 1 for TE polarization. The ODR range is indicated in the figure. One can see that within the ODR range the density of radiation modes is strongly suppressed: The ratio between the radiation and the guided modes is of the order of 10^{-3}. However, the guided-mode density is quite high, even higher than the mode density in bulk material. From this result we conclude that the total mode density is not suppressed at all. For frequencies below the ODR range we observe kinks in the guided- and the radiation-mode contributions. When the frequency is increased, more guided modes will be allowed in the structure, as the layers become optically thicker.[9] A radiation mode will then suddenly cross over to a guided mode; the kinks cancel each other, as can be seen from the absence of kinks in the total density plotted in Fig. 1. For small frequencies, i.e., long wavelengths, the structure behaves as if it is progressively optically thinner, so the total density of states approaches that in air in the limit $\omega \to 0$.

In Fig. 2 the results for TM polarization are shown. Here the picture is quite different. Before and within the ODR range, the contribution of the guided modes is much smaller than in the case of TE polarization. However, this smaller contribution is not due to the PBG. Rather, it is due to the fact that for TM

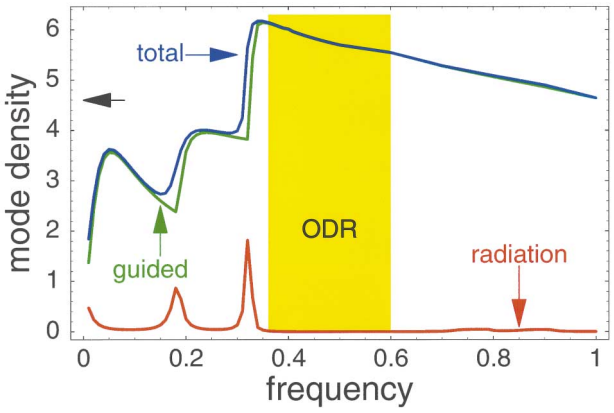

Fig. 1. Mode density versus frequency ω [in c/d_1] for TE polarization for radiation modes, guided modes, and the sum of both (total).

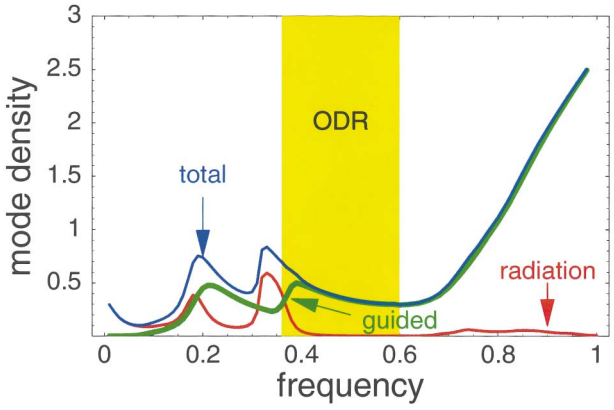

Fig. 2. Mode density versus ω [in c/d_1] for TM polarization for radiation modes, guided modes, and the sum of both (total).

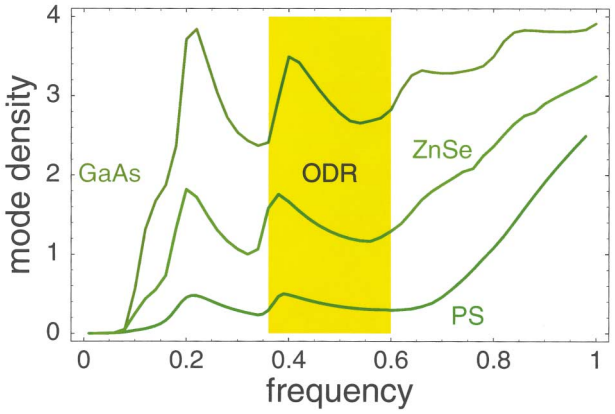

Fig. 3. Mode density versus ω [in c/d_1] for TM-polarized guided modes only, for configurations with layers of PS, ZnSe, or GaAs.

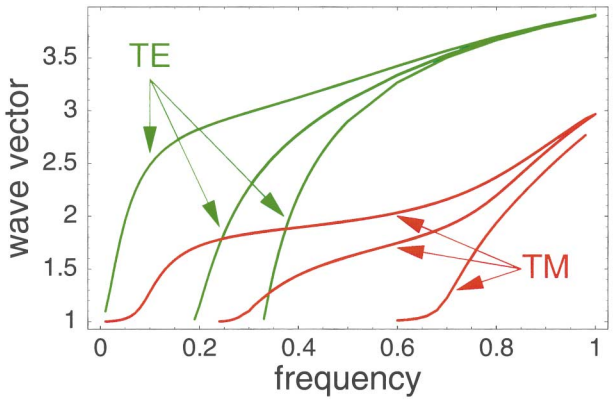

Fig. 4. Wave vector k of the guided mode versus frequency ω [in c/d_2] for the 0th, 2nd, and 4th modes for TE and TM polarization.

polarization the EM field has a component in the direction of periodicity. This component induces a dipole covering on the interface, which hampers confinement and also forces the field into regions with lower dielectric constants, thus preventing buildup of guided modes. For low frequencies, i.e., long wavelengths, this prevention of buildup means that energy is forced outside the structure into the surrounding air, since the structure is optically thin. Only as the layers become optically thicker, i.e., more than $\lambda/4$, does the density of states inside the structure build up properly. From waveguide theory it is already known that the waveguiding performance of TM waves is reduced relative to that of TE waves, owing to an extra term in the TM wave equation,[10] but the effect is quite dramatic in the present case.

In Fig. 3 we plot the total density of TM guided modes as a function of frequency for three different configurations of the multilayer structure. The original PS layer has an index of 1.6, which we replaced with a material with a refractive index of 2.6 (ZnSe) and a material with a refractive index of 3.6 (GaAs). In all cases the density of modes does not immediately build up. However, with decreasing contrast the buildup of density shifts to lower frequencies.

In Fig. 4 we plot the k values of the different guided modes as a function of frequency (these values were input into the calculation of the guided-mode density). $k^{\mathrm{TE}}(\omega)$ has a large derivative when it first appears, whereas the derivative is observed to vanish for $k^{\mathrm{TM}}(\omega)$; this is why steplike behavior is absent from the TM modes.

We have shown that the omnidirectional mirror is an excellent waveguide in the omnidirectional reflection range, especially for TE-polarized modes, as there is almost no emission into radiation modes. Also, the mode density is not dramatically altered; it is as high as would be expected for an ordinary waveguide. Thus, on the inside the crystal bears no signature from the stop band, other than directional change. We have demonstrated rigorously that an outside observation of ODR in a certain frequency range does not mean that the mode density is zero or even small in that frequency range on the inside. Real-life photonic crystals will inevitably be finite, and a proper test of any candidate photonic bandgap material should include both outside testing of ODR and a thorough analysis of the inside mode density in the finite configuration.

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