

Fifty years of Anderson localization

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What began as a prediction about electron diffusion has spawned a rich variety of theories and experiments on the nature of the metal-insulator transition and the behavior of waves—from electromagnetic to seismic—in complex materials.

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Very few believed [localization] at the time, and even fewer saw its importance; among those who failed to fully understand it at first was certainly its author. It has yet to receive adequate mathematical treatment, and one has to resort to the indignity of numerical simulations to settle even the simplest questions about it.

—Philip W. Anderson, Nobel lecture,
8 December 1977

The study of the conductance of electrons belongs to the very heart of condensed-matter physics. The classical Drude theory of electronic conductivity was built on the idea of free electrons scattered by positive ions in metal lattice sites. A key concept in that description was the mean free path, the average length an electron travels before it collides with an ion. According to classical theory, the electronic conductivity should be directly proportional to the mean free path, which experiment had established as large in metals—around 100 nm, some two orders of magnitude larger than the lattice constant.

Physicists had to wait for the discovery of quantum mechanics to understand why electrons apparently do not scatter from ions that occupy regular lattice sites: The wave character of an electron causes the electron to diffract from an ideal crystal. Resistance appears only when electrons scatter from imperfections in the crystal. With that quantum mechanical revision, the Drude model can still be used, but in the new picture an electron is envisaged as zigzagging between impurities. The more the impurities, the smaller the mean free path and the lower the conductivity.

Will any increase in the degree of lattice disorder lead to just a decrease in the mean free path and thus to a lower conductivity, or might something unusual happen along the way? That question was raised a half century ago by Philip Anderson, pictured in figure 1. Beyond a critical amount of impurity scattering, he discovered, the diffusive, zigzag motion of the electron is not just reduced, it can come to a complete halt.¹ The electron becomes trapped and the conductivity vanishes.

To appreciate how that's possible, at least intuitively, one can start with elementary wave mechanics. Suppose an electron propagates in a disordered medium from point A to

point B. One has to sum the amplitudes of all possible paths and square the complex-valued end result to obtain the total probability that the electron arrives at B. That probability consists of a sum of squares—the classical, incoherent contribution—plus many cross-product, interference terms. One could argue that in disordered media, the phases of the interference terms are so random that their sum vanishes on average. That assumption would bring us back to the diffusion model of metallic conductance.

But that neglect of interferences is not always justified. Imagine a wave that travels from point A along a random path to point B and then goes back to A. In figure 2, two possible paths are depicted: a randomly chosen path and the same path traversed in the opposite sense. The two paths interfere constructively and should be treated coherently—that is, summed before being squared. The probability of the electron's returning to A is then twice as large as it would have been if probabilities were added by first squaring and then summing. The enhanced backscattering, known as weak localization, lowers the conductance between A and B by increasing the electron's likelihood of returning to its starting point. But can it eventually localize the electron around A?

Metal-insulator transition

Anderson had been confronted with experiments performed by George Feher's group at Bell Labs—experiments that showed anomalously long relaxation times of electron spins in doped semiconductors. The concept of localized electrons could explain the observation but would break with the conventional diffusion picture. To explain the effect, Anderson used a tight-binding model of an electron in a disordered lattice; at each lattice site an electron feels a random potential and is allowed to tunnel between nearest neighbor sites with a constant rate (see figure 3).

Electrons are waves, of course. But rather than thinking of conduction electrons as extended plane waves with short lifetimes and small mean free paths, one should instead view them as standing waves that are confined in space and thus have long lifetimes. Moreover, not just one or two electrons are localized by a random well in the landscape of the random potential energy; nearly all conduction electrons be-

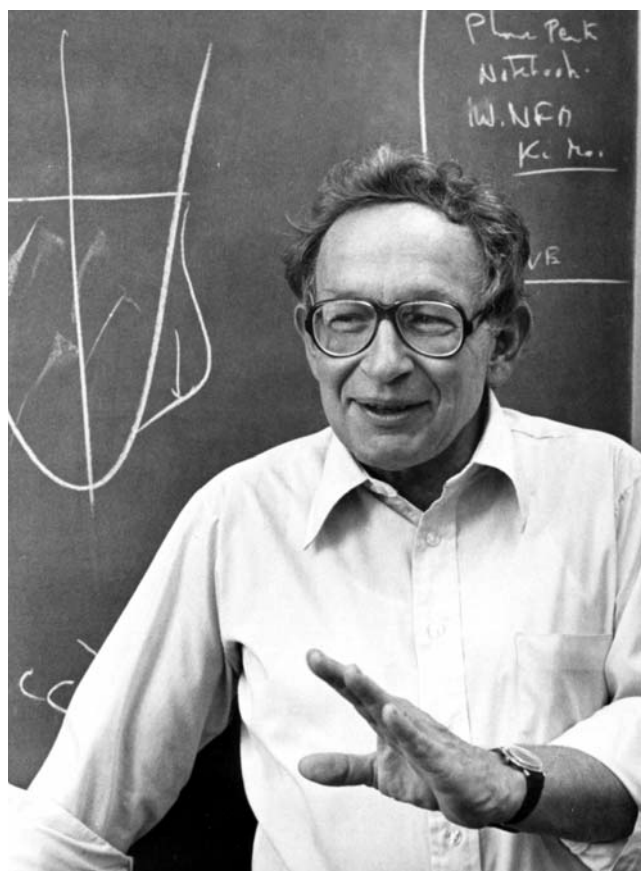


Figure 1. Philip Anderson shared the 1977 Nobel Prize in Physics with Nevill Mott and John van Vleck for work on the electronic structure of magnetic and disordered systems. (Courtesy of the AIP Emilio Segrè Visual Archives, PHYSICS TODAY Collection.)

come localized in concert. For each electron, the multiple scattering events add to cancel each other.

Anderson's model ushered in a new quantum mechanical view of metal-insulator transitions. And in the early 1960s, Nevill Mott introduced the notion of a mobility edge that separates extended and localized states (see his article in *PHYSICS TODAY*, November 1978, page 42). Electrons with large energies—and correspondingly small de Broglie wavelengths—behave conventionally and support an electronic current, Mott argued. But electrons with energies near the band edge, with correspondingly large de Broglie wavelengths, would be localized. The idea of a mobility edge would develop into one of the most studied concepts of condensed-matter physics. For their work on disordered systems, Anderson, his thesis adviser John van Vleck, and Mott shared the 1977 Nobel Prize in Physics.

Ironically, Anderson's 1958 paper hardly got noticed at first; it was cited just 30 times in the first 10 years. Today, it's been cited over 4000 times, though too often as an "unrecognizable monster"² as described by its creator in 1983. Indeed, theoreticians have found a variety of ways to look at localization—from scale-dependent diffusion and fractal wavefunctions to quantum chaos, dense-point spectra, and kicked rotors.

Initial challenges and disputes

The original 1958 work demonstrated that Anderson localization, like any other phase transition, strongly depends on the dimension of the medium. Shortly afterward, theoreticians established that under broad conditions, all quantum states in one-dimensional disordered systems—thin wires, say—are localized. That's counterintuitive, especially when the kinetic energies of electrons generally exceed typical fluctuations in potential energy.

Mott and W. D. Twose usually get the credit for the discovery, although work by M. E. Gertsenshtein and V. B. Vasil'ev already existed in 1959. To say that all states are localized in thin wires does not, however, mean that the conduction of a wire is always small. In 1994 John Pendry showed that in a wire of finite length, a small chance exists that two or more localized modes couple to form a "necklace state" that leads to full transmission of electrons.³

Localization in higher dimensions was much harder to solve. One had to calculate all possible paths that a particle could take. In 1958 Anderson was unable to include "loops," electron paths that eventually come back to the same lattice site. With his postdoc Ragi Abou-Chakra and David Thouless he solved the problem 15 years later by considering an electron hopping randomly on a Cayley tree, a somewhat artificial but useful fractal model in which it's impossible for an electron to return to the same lattice site except by retracing exactly the same path. Although the fractal model erroneously predicted a phase transition in 2D metals, it did allow researchers to analytically solve, for the first time, Anderson localization in higher dimensions and confirmed Anderson's conjecture that loops are, in fact, not crucial to localize the electron.

The complex physics behind Anderson localization has also prompted many disputes. Early on, Mott reasoned that the mean free path ℓ of a conducting electron could never be smaller than the lattice constant a ; that reasoning gives rise to a nonvanishing electronic conductivity. The wavelength λ of conducting electrons at the Fermi surface is of order $2\pi a$. That makes Mott's minimal mean free path similar to the one derived independently in 1960 by Abram Ioffe and Anatoli Regel, $\ell \approx \lambda$, though a factor of 2π smaller. There's not much left to "wave" anymore for a wave whose mean free path has become shorter than its wavelength.

Unfortunately, the Mott minimum disagrees with the now widely accepted scaling theory of localization published in 1979 by the "gang of four"—Elihu Abrahams, Anderson, Donald Licciardello, and T. V. Ramakrishnan.⁴ Never settled, the controversy was explicitly mentioned by the Nobel committee in 1977 and Mott defended his position until his death. Nevertheless, the Ioffe-Regel criterion survived the controversy and was later generalized to more complex media. It is usually cast in the form $k\ell \approx 1$, where the wave vector $k = 2\pi/\lambda$.

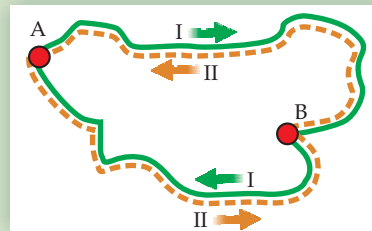


Figure 2. Interference effects influence the propagation of waves in a disordered medium. If two waves follow the same path from A to B, one going clockwise and the other counterclockwise, they interfere constructively on returning to A.

Scaling of transport parameters

The scaling theory of localization gives deep though qualitative insight into how the localization transition appears in finite, open media. It predicts that close to the mobility edge, the conductivity of a material depends on its size, as outlined in figure 4. That prediction has far-reaching consequences. For an electron, it means that the diffusion constant is size dependent.

Inspired by the work of J. T. Edwards and Thouless,⁵ scaling theory puts forward a dimensionless scale parameter g that governs that size dependence. Edwards and Thouless defined the parameter as the ratio between two time scales—the Heisenberg time and the Thouless time—and showed the ratio to essentially be a measure of the conductance. The Thouless time is the time it takes for a conducting electron inside the sample to arrive at the boundary through its zigzag motion, whereas the Heisenberg time is the longest time that an electron wavepacket can travel inside a finite size sample without visiting the same region twice. When the Thouless time exceeds the Heisenberg time, a wavepacket is unable to reach the boundaries and is localized inside the sample. That Thouless criterion for Anderson localization thus asserts that states are localized when $g < 1$. The criterion also turned out to have universal validity: Scaling theory adopted the dimensionless conductance g as its only parameter.

Among the theory's predictions is the existence of two critical exponents. One is the exponent with which the conductivity vanishes with energy as the mobility edge is approached; the other governs the divergence of the localization length—the typical size of a localized wavefunction—below the mobility edge. During the 1970s, computer simulations were still rare in the field. But despite Anderson's pessimism in 1977, precise values for the exponents were soon calculated by numerical studies.⁶

Experiments with electrons

Unfortunately, electron localization was devilishly hard to confirm. Around 1983 Mikko Paalanen and Gordon Thomas published conductivity measurements around the metal-insulator transition of 3D doped charge-uncompensated silicon. The critical exponents were equal to 0.5 on both sides of the transition. Charge-compensated semiconductors, in contrast, were observed to have a critical exponent close to 1. Numerical work⁶ had predicted an exponent larger than $\frac{1}{2}$. The discrepancy prompted what became known as the “exponent puzzle.” Much later, in 1999, researchers argued that an exponent of 1 is recovered in the experiments on silicon if the conductivity is correctly extrapolated to zero temperature.

In 1988, Aart Pruisken established the connection between Anderson localization and the integer quantum Hall effect.⁷ Eight years earlier Klaus von Klitzing's team had observed that the Hall conductance of a 2D electron gas exhibited plateaus. The conductance is constant with magnetic field but suddenly rises when the Fermi energy of the conducting electrons approaches the Landau levels—the quantized cyclotron orbits of electrons around magnetic field lines. The quantum Hall effect could be explained if the electrons were extended near the Landau levels but localized elsewhere. Pruisken's team found a magnificent opportunity to test scaling theory. Thermal processes affect the phase of the electrons and restrict their quantum coherence over a finite length. So, by changing the temperature, one changes the sample size explored by the electrons. The team observed that the transition between the plateaus exhibited a temperature dependence in beautiful agreement with scaling theory.

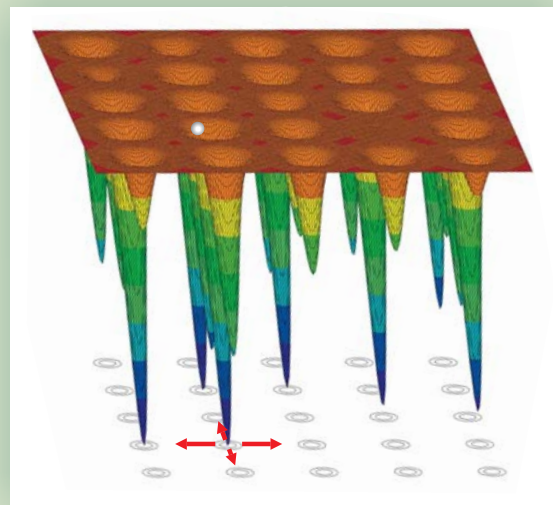


Figure 3. The Anderson model. Imagine an electron (silver) hopping on a two-dimensional lattice with random potential energies at each site. Quantum mechanics allows the electron to tunnel from one site to another through large energy barriers as depicted by the red arrows. The electron's energy thus changes randomly, although at each lattice site the spatial extent of its wavefunction (sketched below the potential) is assumed constant, leading to a constant tunneling rate. On an ordered lattice with all wells the same depth, the electron would be completely mobile for a range of energies. But here, a critical amount of randomness in the well depths localizes the electron, although on a scale larger than the lattice constant. For another perspective of what occurs as a lattice changes from perfect to disordered, see this month's cover.

Weak localization

In the early 1980s a genuine microscopic theory for localization in 3 dimensions did not exist and no one knew how the size dependence of conductance would emerge on a microscopic scale. Experimental work indicated the existence of weak localization—the enhanced backscattering of electron waves discussed above, now often seen as a precursor to Anderson localization. Based on that work, Wolfgang Götzke, Dieter Vollhardt, and Peter Wölfle formulated what became known as the self-consistent theory, which revealed how conventional multiple scattering of extended waves breaks down to make way for Anderson localization,⁸ something that had always been questioned by experts, including Anderson himself.⁹

Classical waves

The self-consistent theory and a 1986 observation of the weak localization of light by the groups of Akira Ishimaru, of Georg Maret, and of one of us (Legendijk) set the stage for a search for Anderson localization using classical waves such as light and sound. Sajeev John had already predicted the existence of a frequency regime in which electromagnetic waves are localized.¹⁰ The question was, at what frequency should the transition occur? By applying the self-consistent theory of electron localization to classical waves, Costas Soukoulis, Ping Sheng, and their colleagues were able to make precise predictions about where to look.¹¹

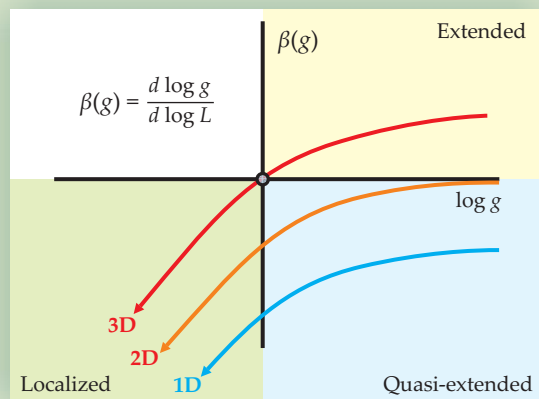


Figure 4. According to scaling theory, Anderson localization is a critical phenomenon, at least in three dimensions. The scaling function $\beta(g)$ describes how—or more precisely, with what exponent—the average conductance g grows with system size L . For a normal ohmic conductor in D dimensions, the conductance varies as L^{D-2} ; consequently, $\beta(g) \sim D - 2$ for large g . Thus the beta function is positive for three-dimensional conductors, zero for two-dimensional conductors, and negative in one dimension. In the localized regime, g decays exponentially with sample size so that $\beta(g)$ is negative. In three dimensions, that leads to a critical point at which β vanishes for some special value for g associated with the mobility edge. Lower-dimension systems do not undergo a genuine phase transition because the conductance always decreases with system size. A small 2D conductor, for instance, will look like a metal in the quasi-extended regime, but all its states are eventually localized if the medium is large enough.

Classical waves offer certain advantages for studying localization. Unlike electrons, photons don't interact with each other, and wave experiments are easy to control experimentally at room temperature; frequency takes over the role of electron energy. One drawback of classical waves, however, is that they do not localize at low frequencies, where the mean free path becomes large due to weak, Rayleigh, scattering.

To recognize whether incoming classical waves are localized in a material, one could examine how the transmission scales with system size. In regular diffusive systems, the transmission is dictated by Ohm's law, in which the signal intensity falls off linearly with thickness. In the regime of Anderson localization, the transmission should decay exponentially with length. However, one should be careful to exclude absorption effects, which also show up as exponential decay.

A huge advantage of using classical waves is that other properties in addition to conductance—for example, the statistical distribution of the intensity, the complex amplitude of the waves, and their temporal response—can be measured. All those properties are expected to be strongly influenced by localization. In particular, the localized regime is predicted to exhibit large, non-Gaussian fluctuations of the complex field amplitude and long-range correlations in the intensity at different spots or at different frequencies.

Light

Anything translucent scatters light diffusively. Think, for instance, of clouds, fog, white paint, human bones, sea coral, and white marble. For those and most other naturally disordered optical materials, the scattering strength is far from that required for 3D Anderson localization. Systems that scatter more strongly can be synthesized, though. For example, material can be ground into powder, pores etched into solids, and microspheres suspended in liquids (see figure 5).

For years researchers have worked with titania powder that is used in paints for its scattering properties. Thanks to the powder's high refractive index (about 2.7) and submicron grain size, mean free paths are on the order of a wavelength. Experiments reveal clear signs in the breakdown of normal diffusion. To observe localization the challenge is to maximize the scattering without introducing absorption.

One way is to use light whose frequency is less than the electronic bandgap of a semiconductor so that it cannot be absorbed but whose refractive index is still high. In 1997, two of us (Wiersma and Lagendijk) and coworkers ground gallium arsenide into a fine powder and observed nearly complete localization of near-IR light, as deduced from scale-dependent diffusion that was measured.¹² Two years later

Frank Schuurmans and coworkers etched gallium phosphide into a porous network. With a mean free path of only 250 nm, it is, to date, the strongest scatterer of visible light.

The scale dependence of diffusion is also studied using time-resolved techniques in which the material is excited by a pulsed femtosecond source. The time evolution of the optical transmission can be measured down to the one-photon level. As time increases, so does the sample size explored by the waves. Scale-dependent diffusion may lead to a time-dependent diffusion constant. As a result, the transmission intensity should fall off at a slow, nonexponential rate. In 2006 Maret's group measured time tails up to 40 ns in titania powders that had surprisingly large values for the mean free path ($k\ell \approx 2.5$); they found just such a nonexponential time decay in transmission.¹³

Microwaves

At the millimeter wavelengths of microwaves, it's relatively easy to shape individual particles, such as metal spheres, that scatter strongly. By randomly placing the spheres in a tubular waveguide with transverse dimension on the order of a mean free path (typically 5 cm), one can study the statistics of how the microwave field fluctuates. The quasi-1D geometry of the system—essentially a thick wire or multimode fiber—is advantageous because many theoretical predictions become relevant, mostly from the DMPK theory. That theory owes its name to its founders—Dorokhov, Mello, Pereyra, and Kumar—and takes arguments from chaos theory to make precise predictions about the full statistical properties of a wire's transmission when its length exceeds the localization length.

The onset of localization is again governed by the dimensionless conductance g , which is here essentially equal to the ratio of the localization length and the sample length. Using microwaves, Azriel Genack and colleagues have explored a broad range of g values, including the localized regime $g < 1$. Indeed, their observations of anomalous time-dependent transmission, scale-dependent diffusion, large fluctuations in transmission, and long-range correlations of both the intensity and the conductance of microwaves have led to a rich and complete picture of Anderson localization in thick wires.¹⁴ Statistics, their work illustrates, can reveal the onset of localization even in the presence of optical absorption.

Acoustics

Ultrasound is particularly well-suited for time-dependent localization studies because of the long times over which energy can be monitored. As early as 1990, using inhomogeneous 2D

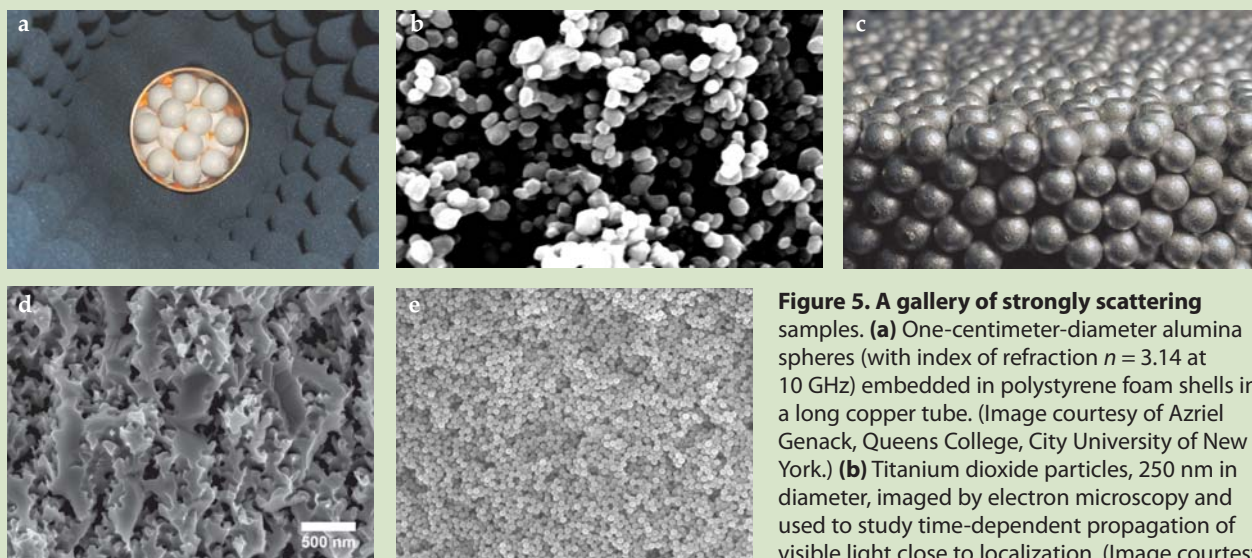


Figure 5. A gallery of strongly scattering samples. (a) One-centimeter-diameter alumina spheres (with index of refraction $n = 3.14$ at 10 GHz) embedded in polystyrene foam shells in a long copper tube. (Image courtesy of Azriel Genack, Queens College, City University of New York.) (b) Titanium dioxide particles, 250 nm in diameter, imaged by electron microscopy and used to study time-dependent propagation of visible light close to localization. (Image courtesy of Georg Maret, University of Konstanz, Germany.)

(c) An elastic network of 4-mm aluminum beads brazed together. (Image courtesy of John Page, University of Manitoba.) (d) Porous gallium phosphide, etched in diluted sulphuric acid and imaged by electron microscopy. The pore size is optimized to scatter visible light. (e) Gallium arsenide powder with $n \approx 3.5$ and an average particle size of $1 \mu\text{m}$. With this material one can observe scaling of the mean free path in transmission measurements.

plates, Richard Weaver observed around a point source a genuine concentration of energy whose intensity slowly decayed due to absorption.¹⁵ In a classical picture, waves that survive absorption can still interfere.

In 2008 John Page, Sergey Skipetrov, and coworkers reported Anderson localization of ultrasound in a 3D elastic network of aluminum beads.¹⁶ Using a pointlike source of ultrasound energy at the entrance of the sample, the group measured how the elastic energy expands in transverse directions. In conventional diffusive samples, that expansion would grow with the square root of time, behavior reminiscent of a Brownian random walk. The transverse confinement of elastic energy is thus a direct consequence of sound localization. As shown in figure 6, the transverse intensity pattern measured across the output surface of the sample illustrates how complex the spatial structure of localized states can be.

Photonic bandgap materials

A photonic crystal is a periodic lattice that diffracts light much like a semiconductor diffracts electrons. Thanks to Bragg reflection, transmission is forbidden for certain wavelengths and directions. Several challenges are being pursued, among them the confinement of light in microcavities, the guiding of light with unusual dispersion, and the creation of materials that suppress spontaneous emission.

Most, if not all, photonic crystals exhibit structural disorder to some extent and thus scatter light. In 1984 Sajeev John predicted the existence of localized states near the band edges of the spectrum, much like the localized electron states that occur near the band edges of doped semiconductors. In a photonic crystal light is easier to localize because its propagation in certain directions is already hindered; John argued that even a modest amount of disorder is sufficient to do the job. (See his article in *PHYSICS TODAY*, May 1991, page 32.)

To date, Anderson localization has never been observed in 3D photonic bandgap materials, although several experimental efforts are under way. Two years ago a group at the

Technion–Israel Institute of Technology in Haifa reported a related phenomenon—transverse localization of light in a 2D bandgap material.¹⁷ Mordechai Segev and coworkers designed an experiment to localize a wavepacket along two transverse directions while it continued to propagate along the third. Based on a prediction from Lagendijk in 1989, Segev’s experimental realization meant that researchers could measure localization in space rather than deduce it from a transmission spectrum (see *PHYSICS TODAY*, May 2007, page 22).

Interactions

The localization problem becomes more complex if one goes beyond the picture of noninteracting particles. The possibility that repulsive interactions between electrons could destroy localization was already a worry in the early 1960s. As for the interaction between localized electrons and phonons, Mott had considered a model in which thermally excited lattice vibrations provide electrons with the necessary activation energy to jump between localized states that are close in energy but spatially distant. That “variable-range hopping” leads to a stretched exponential dependence of the electric conductivity on temperature and was widely observed in doped semiconductors and amorphous metallic compounds. The success of Mott’s model even prompted the question of whether the phonons are, in fact, required to provide the activation energy. Perhaps interactions between the electrons themselves could explain the thermally induced electron conductivity in the localized regime.

The first answers came from the work of Larry Fleishman and Anderson in 1980. At low enough temperatures, they argued, repulsive interactions neither destroy the localized electronic states nor induce thermally excited hopping. Conductance should still vanish at low temperatures. Around the same time, Boris Altshuler and coworkers found that interactions between electrons destroy the constructive interferences and thus lead to a finite, almost diffusive con-

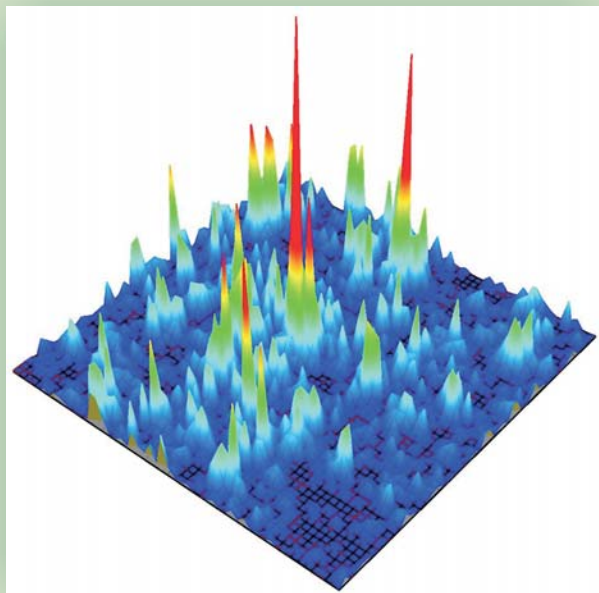


Figure 6. The energy density of 2.4-MHz elastic waves localized in the network of aluminum beads pictured in figure 5c. In the small, isolated hot spots in the intensity of the wavefunction, the transmitted wave energy is more than 40 times the ensemble average. The length of each axis is about 15 mm. (Image courtesy of John Page, University of Manitoba.)

ductance. Recent work by Denis Basko, Altshuler, and colleagues combined the two results and concludes that repulsive interactions, together with disorder in the potential energy landscape, lead to a metal-insulator transition at some intermediate, finite temperature.¹⁸

Cold atoms and beyond

When atoms are cooled to near absolute zero temperature, their de Broglie wavelength becomes large—fractions of a micron. Research groups in Palaiseau, France, and Florence, Italy, recently observed that the expansion of ultracold atoms in a disordered 1D potential can be halted—the first evidence for Anderson localization of atomic gases in one dimension. An optical interference pattern generates the random potential from which atoms scatter. The advantage of cold atoms over electrons is that their interactions, repulsive and attractive, can be tuned. Three-dimensional localization was recently observed by another French collaboration using “kicked” cold atoms. The experiment, performed by Julien Chabé and colleagues in 2008, confirmed a one-parameter scaling around a mobility edge and found critical exponents consistent with the 3D Anderson model. For details on the cold-atoms approach to localization, see the companion article by Alain Aspect and Massimo Inguscio on page 30 of this issue.

After more than a half century of Anderson localization, the subject is more alive than ever. The role of interactions in electron localization is still not well understood and several groups are now pursuing classical wave localization. Speculations already exist about the localization of seismic waves; Earth’s volcanic regions may be good places to look since the mean free path and wavelength of seismic waves are comparable in magnitude. The lesson of history, though, is that localization often shows up at unexpected places and in unexpected disguises.

We thank Alain Aspect, Denis Basko, Philippe Bouyer, Dominique Delande, Azriel Genack, François Germinet, Massimo Inguscio, Georg Maret, John Page, Michael Schreiber, Sergey Skipetrov, David Thouless, and Peter Wölfle for their support in writing this article.

References

1. P. W. Anderson, *Phys. Rev.* **109**, 1492 (1958).
2. P. W. Anderson, in *Localization, Interaction, and Transport Phenomena: Proceedings of the International Conference, August 23–28, 1984, Braunschweig, Fed. Rep. of Germany*, B. Kramer, G. Bergmann, Y. Bruynseraede, eds., Springer, New York (1985).
3. J. B. Pendry, *Adv. Phys.* **43**, 461 (1994).
4. E. Abrahams, P. W. Anderson, D. C. Licciardello, T. V. Ramakrishnan, *Phys. Rev. Lett.* **42**, 673 (1979).
5. J. T. Edwards, D. J. Thouless, *J. Phys. C* **5**, 807 (1972).
6. B. Kramer, A. MacKinnon, *Rep. Prog. Phys.* **56**, 1469 (1993).
7. A. M. M. Pruisken, *Phys. Rev. Lett.* **61**, 1297 (1988).
8. D. Vollhardt, P. Wölfle, *Phys. Rev. Lett.* **48**, 699 (1982).
9. P. W. Anderson, in *Ill-Condensed Matter: Les Houches 1978, Session XXXI*, R. Balian, R. Maynard, G. Toulouse, eds., North-Holland, New York (1979), p. 162.
10. S. John, *Phys. Rev. Lett.* **53**, 2169 (1984).
11. E. N. Economou, C. M. Soukoulis, *Phys. Rev. B* **40**, 7977 (1989); P. Sheng, Z.-Q. Zhang, *Phys. Rev. Lett.* **57**, 1879 (1986).
12. D. S. Wiersma, P. Bartolini, A. Lagendijk, R. Righini, *Nature* **390**, 671 (1997).
13. M. Störzer, P. Gross, C. M. Aegerter, G. Maret, *Phys. Rev. Lett.* **96**, 063904 (2006).
14. A. A. Chabanov, M. Stoytchev, A. Z. Genack, *Nature* **404**, 850 (2000).
15. R. L. Weaver, *Wave Motion* **12**, 129 (1990).
16. H. Hu, A. Strybulevych, J. H. Page, S. E. Skipetrov, B. A. van Tiggelen, *Nat. Phys.* **4**, 945 (2008).
17. T. Schwartz, G. Bartal, S. Fishman, M. Segev, *Nature* **446**, 52 (2007).
18. D. Basko, I. L. Aleiner, B. L. Altshuler, *Ann. Phys. (N.Y.)* **321**, 1126 (2006). ■